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A REINVESTIGATION OF THE DECAY OF Na-22*

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ABSTRACT. The amount of positron emission in the decay of Na-22 has been determined to be 0.899 ± 0.003 using a 4π plastic beta scintillation spectrometer in conjunction with double and triple coincidence techniques. This leads to an $\varepsilon/\beta+$ ratio of 0.112 ± 0.004 which is slightly better than the best earlier result of Sherr and Miller. Comparison of the measured value with the theoretical ratio of 0.1135 ± 0.002 leads to a value for the Fierz interference term $b_{GT} = -0.004 \pm 0.012$, showing the extreme smallness of the cross term in allowed Gamow-Teller transitions. In the appendix a brief summary of the status of the Fierz term as revealed by our studies and other work is presented.

INTRODUCTION

2.6 year Na-22 decays by positron emission and electron-capture to the firs texcited state of Ne-22 at 1.28 MeV followed by a gamma ray of this energy to the ground state. The decay scheme is shown in figure 1. The spin of Na–22 has been measured to be 3 (Mack, 1950). The spin of the 1.28 MeV state of Ne-22 is 2^+ from systematics of even-even nuclei (Scharff-Goldhaber, 1953). Hence the transition $3^+ \rightarrow 2^+$ follows the selection rule $\Delta J = 1$, No and is therefore pure Gamow-Teller. The electron-capture to positron branching ratio in the decay of Na–22 has been extensively studied. A summary of previous work is given in a very recent paper by Konijn *et al* (1959). So far the best reported value is that of Sherr and Miller (1954) who obtained $\epsilon/\beta^+ = 0.110 \pm 0.006$ by an elegant experiment, and by comparing with the theoretical value of 0.1135 ± 0.0020 estimated the Fierz interference term to be $(-1 \pm 2)\%$. Since all the present interpretations of Fierz interference are based on this experiment we have been prompted to attempt a more precise determination of this ratio, if possible.

The principle of the present experiment is extremely simple. If we have a gamma counter which detects the 1.28 MeV gamma ray and a beta

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counter which detects the positrons, then assuming that all the positrons are counted we can write for the beta-1.28 coincidences

$$N_{\beta^+, 1\cdot 28} = N_0 f_+ \sigma_{1\cdot 28} = a$$

and for the gamma ray

$$N_{1\cdot 28} = N_0 \sigma_{1\cdot 28} = b$$

where N_0 is the transition rate,

$f_+ =$ the fraction of decays bypositron emission

and $\sigma_{1\cdot 28} =$ the efficiency for detecting the1 28 MeV gamma ray. The ratio of a to b then yields f_+, from which the ϵ/β^+ ratio can be computed. This is possible provided the entire positron spectrum can be measured.

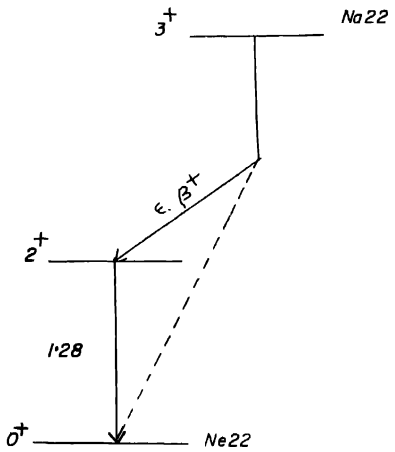

Fig. 1. Decay Scheme of Na–22

We have employed a 4π plastic scintillation counter for detecting the positrons and a NaI(Tl) counter for the 1.28 MeV gamma ray. The gamma counter is biased to accept only the photopeak. The effectiveness of the 4π plastic scintillation counter for measuring the shapes of beta spectra has been demonstrated by the work of Johnson *et al* (1956) and more recently by Robinson and Langer (1958), and is substantiated by the present experiments.

A Na–22 source from a HCl solution was evaporated on a 0.0001″ mylar foil and covered with a similar foil. The 4π counter was formed in the following way. Two plastic cylinders, each 3mm thick and 1 cm in diameter were chosen. One of the cylinders had a depression 1/2 mm deep and 1/2 cm in diameter. The Na–22 sandwich was placed in the depression. The two cylinders were pressed together to form the 4π counter. A cone-shaped light pipe 1-1/2″ long having a well at the apex was mounted on a DuMont 6292 phototube. To the bottom of the well the 4π plastic scintillation crystal was cemented by means of Canada balsam. The sides of the well had been painted white to ensure good light collection. The top of the well had a thin aluminium foil whitened inside. The gamma counter was a 2″ cube NaI(Tl) crystal which had a resolution of 11% for 0 661 MeV gamma ray of Cs-137. The 4π counter had a resolution of 16% for the 0.624 MeV K-conversion line of Cs-137. The entire assembly of crystal and counters was surrounded by 2″ of lead at 4″.

EXPERIMENTAL

The general features of the 4π counter were investigated by a P-32 source using plastic cylinders, each 5mm thick and 1 cm in diameter. A Fermi plot of

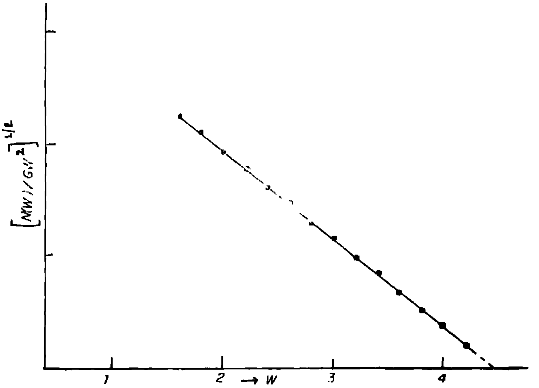

Fig. 2. Fermi plot of P-32 spectrum taken with a 4π plastic scintillation spectrometer. Note the end-point at 1.72 MeV.

the spectrum is shown in figure 2. An end-point of 1.72 MeV is indicated, in good agreement with the value in the literature (King, 1954). Experiments on Na–22 were started with plastics of dimensions described in the Introduction The gamma counter was set on the photopeak of the 1.28 MeV gamma ray. The peak had a width of 3.5 volts at 35 volts. This was used to gate the 20-channel analyzer. The positron spectrum coincident with the 1.28 MeV gamma ray is shown in figure 3. Energy calibration of the spectrometer was obtained by using external gamma rays of Na-22(0.511 MeV), and Cs-137(0.661 Mev). The Compton edges located at 3/4 of the maximum were used. The calibration is also shown in figure 3(c). The calibration curve intercepted the axis corresponding to zero pulse height at 18 KeV in agreement with similar observations by Johnson *et al* (1956).

Because of the fact that the plastic chosen had dimensions somewhat greater than the range of positrons, one would expect that the observed beta spectrum

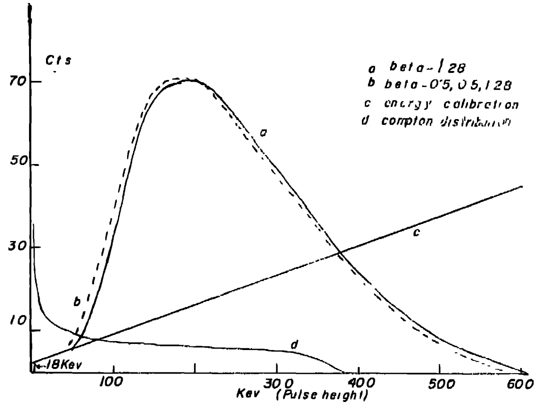

Fig. 3.

(*a*) Continuous curve—beta spectrum of Na–22 in 4π plastic counter coincident with the 1.28 MeV gamma ray.

(b) Dotted curve—beta spectrum of Na–22 in triple coincidence with the 1.28 MeV gamma ray and the two annihilation quanta. The spectrum is normalized to the doubles spectrum (beta–1 28*) above 50 KeV to 1/10%.

(c) Energy calibration of the 4π plastic counter using Compton edges of 0.511 (Na-22) and 0.661 (Cs-137) MeV.

(d) Compton distribution of Na–22 gamma rays in 4π plastic counter (with positrons completely stopped by lucite) coincident with the annihilation radiation and the 1.28 MeV gamma ray.

may not be the correct one, but somewhat distorted by the simultaneous detection of a beta particle and its associated Compton electron. Thus the effect would be qualitatively to shift the spectrum towards higher energy, without changing the area under the spectrum.

In order, therefore, to obtain the undistorted spectrum, the beta spectrum was measured in triple coincidence with the 1.28 MeV gamma ray and the two annihilation quanta. The experimental arrangement and a functional diagram of the electronic circuitry are shown in figure 4. Pulses from the two 0.511MeV counters and the 1.28 MeV counter were fed to a triple coincidence circuit, whose output was used to gate the 20-channel analyzer. The positron spectrum gated by the triples is shown also in figure 3, normalized to the doubles spectrum beyond 50 KeV. The statistical error for each point on the triples spectrum varied from 2 to 4%. The doubles and the triples spectra are indeed displaced as expected. To obtain a quantitative justification for the spectral displacement, the positrons were completely stopped in just enough lucite and the Compton distribution was obtained in coincidence with the annihilation radiation and the 1.28 MeV gamma ray. The spectrum thus obtained is shown as curve *d* in figure 3 and is seen to be similar to the one that is obtained using an external source except for the absence of edge effects.

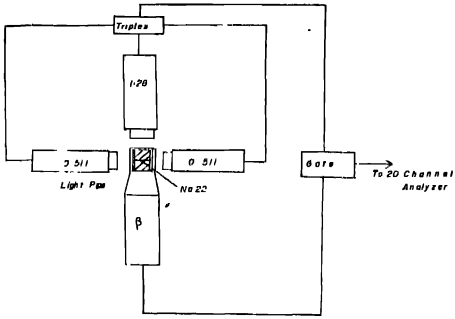

Fig. 4. Block diagram of experimental set-up for Na–22 studies.

If the assertion that the effect of the Compton distribution due to annihilation radiation is simply to shift the doubles spectrum is indeed correct, then it must be possible to express the doubles spectrum $d(h)$ in terms of the triples spectrum $t(h')$, and the Compton distribution $C(h-h')$. In other words, we should be able to write

$$d(h) = \sum^{h} t(h')\ C(h-h')h.$$

A numerical calculation was carried out to test this assumption. For an assumed Compton of 6% the agreement from point to point was 3–4%. The assumption of 6% is not inconsistent with the dimensions of the plastic and the Compton cross-section (the choice of 6% is not critical, since the triples spectrum itself was known to 2-4%). The agreement thus obtained provides quantitative justification for the assertion made earlier.It must be pointed out in this connection that the effect of the inner bremsstrahlung is to displace the true spectrum in a direction opposite to that due to the Compton distribution but because of the weakness of the effect, the Compton effect predominates. The preservation of areas under the doubles and triples spectra is indicated by the fact that the two areas could be normalized to within 1/10%.

Since the lowest energy observed was around 40 KeV, an extrapolation of the spectrum to zero energy has to be made in order to obtain the area under the whole beta spectrum. To do this, the following procedure was adopted. The ideal

Fermi spectrum corrected for screening was plotted. The spectrum was distorted for finite resolution at various points of the spectrum by folding in a gaussian of the proper width. The assumption was made that the half-width varied as the square root of the energy over the entire energy range. Choosing various energies (designated as h_{min}), the area to the right of h_{min} was obtained. It was determined that below 50 KeV the area under the beta-spectrum with and without resolution correction differed only by 1/10% and amounted to 5.3% of the area under the beta-spectrum beyond 50 KeV. Thus the area under the ideal Fermi distribution was taken as the correct area. This when added to the area due to the remaining portion of the doubles spectrum (which had been corrected by the Compton distribution to get the undisplaced spectrum) would give the total area.

In order to test for any possible systematic errors the ratio of area to the right of h_{min} and the entire area from 50 KeV upto the maximum energy was plotted as a function of h_{min} both for the ideal Fermi spectrum corrected for finite resolution, and the actual doubles beta spectrum corrected to the triples spectrum. The result is displayed in figure 5. It is observed that the data of three different runs

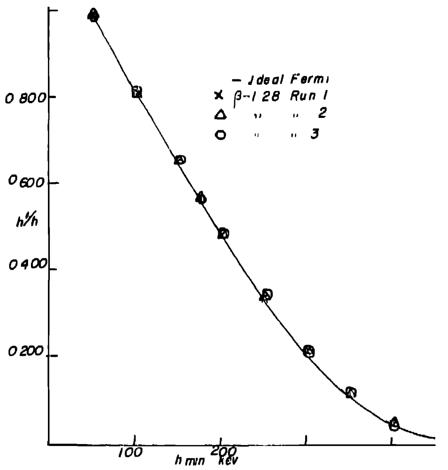

Fig. 5. Study of systematic errors in the Na–22 experiment.

are consistent within themselves to 1-1/2% and with the theoretical plot to within 1%. This may be taken as evidence for the absence of any systematic errors.

The experiments were repeated with and without shielding. The effect of channel width on the gamma ray side was next studied. A different source was made and the experiments repeated. In each case consistent results were obtained. Throughout the course of the experiments the counters were periodically checked. The energy calibration of the beta spectrometer was carried out before and after each run. The overall statistical error in the double run was 1/10 to 2/10%. Altogether seven runs were made.

RESULTS

The data from six of the runs are assembled in Table I together with explanation. The average value of f_+ is calculated to be 0.899 ± 0.003. This yields an average ϵ/β^+ ratio of 0.112 ± 0.004.

Apart from statistical error, the other uncertainty is due to the folding of the Compton distribution, and in the estimation of the areas under the beta and gamma spectrum. A calculation was made to see how much error would be introduced if the half-width of the gaussian curve deviated from obeying the $\sqrt{E}$ law. Dependences proportional to $E^{0.4}$ and $E^{0.6}$ were investigated. From this it was concluded that the error introduced is less than 1/10% in the final result.

Finally an error in the determination of the end-point of the positron spectrum would introduce an error in the value of h_{min}. Because of the assumed linearity in energy scale, this would tend to introduce a linear systematic error(as distinguished from any due to the apparatus itself). In Table 2 the end-points are tabulated for various runs together with uncertainties. From this table the systematic error introduced in this way is estimated to be less than 1.2%. Thus allowing for this our ϵ/β^+ ratio would at worst become

$$\epsilon/\beta^+ = 0.112 \pm 0.005$$

resulting in the Fierz term (see Discussion)

$$b_G = -0.004 \pm 0.013.$$

DISCUSSION

The computed value of ϵ/β^+ is somewhat better than that of Sherr and Miller. The theoretical value of ϵ/β^+ is 0.1135 ± 0.0020, when corrected for screening and 6.5% L-capture (Rose and Jackson, 1949). The value of $<W^{-1}>$ for W $= 2.061$ for Na-22 is 0.7. The Fierz term is computed from the expression

$$b_{GT} = \frac{R/R_0 - 1}{2[1 + R/R_0 < W^{-1} >]} = -0.004 \pm 0.012.$$

Na-22 is perhaps the ideal case for determining the Fierz term because of the low Z involved. It is very unfortunate that the end-point of the positron spectrum is not known well enough to attempt any further refinement in experimental

TABLE I

Na^{22}(Data)

Run No.	Conditions	(a) $\beta+$, 1.28 (> 50 kev)	(b) $\beta+$, 1 28 (> 0 kev)	(c) $N_{1\cdot28}$ cps	(d) $f_+ = (b)/(c)$
Source No. 1	1. 2″ of Pb shield at 4″. γ-ray ch. width=3.5 volts	64 08±0.18	67.48±0.19	75.21±0.11	0.898±0.003
	2. No shield. ch. width=3.5 volts	64 23±0 19	67 63+0.20	75 24±0.15	0.900±0.003
	3. No shield ch. width=3 volts	57.82±0.22	60 88±0.23	67.76±0 15	0.899±0.004
	4. 2″ of Pb shield at 4″. γ-ray ch. width=3.5 volts	48.31±0.14	50.87±0 15	56 52±0 10	0.900±0 003
No. 2	5. No shield ch. width—3.5 volts	48.07+0 17	50 62±0.18	56 31+0.11	0899±0.004
	6. No shield ch. width=3 volts	42 78±0.13	45.05±0.14	50 11±0 10	0.899±0.003

$\epsilon = 1 - f_+ = 0.101 \pm 0.003$

$\therefore \ \epsilon/\beta^+ = \dfrac{0\ 101 \pm 0.003}{8/899 \pm 0.003} = 0.112 \pm 0.004$

Average value of

$f_+ = 0\ 899 \pm 0.003$

TABLE II

End-point energies of positron spectrum

Run No	End-point (keV)
1	546±11
2	541±10
3	548±11
4	539±10
5	540±10
6	544±11

TABLE III

Summary of results on Fierz term

Nucleus	Transition	W_0	b_{GT}	Reference
Ga–68	$1^+ \to 2^+$	4 70	−0.03±0.02	Ramaswamy, 1959
Co–58	$2^+ \to 2^+$	1.924	−0.004±0.014	,, 1958
Na–22	$3^+ \to 2^+$	2.061	−0 004±0.012	present work

techniques to measure ϵ/β^+ ratio. In any case it has been demonstrated that the plastic scintillation counter can be effectively used in the study of beta spectra and precision results obtained if analyzed with caution.

CONCLUSIONS

A reinvestigation of the electron capture to positron branching ratio in the decay of Na-22 has been made with somewhat greater precision than has been possible before, using a 4π plastic scintillator and a gamma counter in conjunction with double and triple coincidence techniques. The result for ϵ/β^+ ratio is 0.112 ± 0.004. It is suggested that the beta spectrum end-point be measured with greater precision to make much more meaningful estimates of the Fierz term. It would be further of interest to measure K/β^+ ratios in unique forbidden transitions allowed only by Gamow-Teller selection rules.

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APPENDIX

SUMMARY OF FIERZ INTERFERENCE IN BETA DECAY

In this section the conclusions regarding the status of Fierz interference in Gamow-Teller transitions as indicated by our measurements reported here and elsewhere are summarised (Table III).

Gerhart (Gerhart, 1958) has made an excellent analysis of Fierz interference in Fermi transitions and concludes $b_F = 0.00 \pm 0.12$. A brief review of Fierz interference in beta-decay has been recently given by the author(Ramaswamy, 1959). From Table 3 one sees that the best evidence for the smallness of the Fierz term in G-T interaction comes from Na-22. Konijn *et al* have summarized data regarding the Fierz term as determined by the K/β^+ ratio technique. They conclude that $b_{GT} = -0.007 \pm 0.010$.

From evidence presented above and from Gerhart's analysis one can conclude that the Fierz term in allowed transitions is practically zero. Before parity nonconservation was discovered the Fierz term in G–T transition could be expressed as

$$b_{GT} = -\frac{C_A^* C_T}{|C_A|^2 + |C_T|^2}$$

The smallness of b could be interpreted as implying that C_A/C_T or C_T/C_A was small . With the discovery that parity is not conserved in beta-decay, the definition of b has acquired the extended form

$$b_T = \frac{Re\,(C_A C_T^* + C_A' C_T'^*)}{|C_A|^2 + |C_T|^2 + |C_A'|^2 + |C_T'|^2}$$

where

$C_{A,T}$ are the parity conserving and

$C_{A',T'}$ are the parity non-conserving coupling constants.
The *s denote complex conjugation resulting from a possible violation of time reversal invariance.

With the new definition of b, the smallness of b means

$$Re(C_A C_T^* + C_A' C_T'^*) = 0.$$

This implies that

$$\frac{C_A'}{C_A'} = -\frac{C_T'^*}{C_T^*}$$

Nothing more can be said concerning the coupling constants unless the relation between the parity conserving and non-conserving coupling constants is known. It is now established from electron polarization measurements on pure Gamow-Teller transitions that $C_A/C_A' \simeq -1$

Thus the parity conserving and non-conserving coupling constants seem to have about the same strength. The loss of definitiveness of the Fierz term is one of the consequences of the discovery of parity non-conservation.

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